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The 3D inversion of airborne gamma-ray spectrometric data

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SUMMARY

We present a new method for the inversion of airborne gamma-ray spectrometric line data to a regular grid of radioelement concentration estimates on the ground. The method incorporates the height of the aircraft, the 3D terrain within the field of view of the spectrometer, the directional sensitivity of rectangular detectors, and a source model comprising vertical rectangular prisms with the same horizontal dimensions as the required grid cell size. The top of each prism is a plane surface derived from a best-fit plane to the digital elevation model of the earth's surface within each grid cell area.

The method is a significant improvement on current methods, and gives superior interpolation between flight lines. It also eliminates terrain effects that would normally remain in the data with the use of conventional gridding methods.

Key words: gamma-ray spectrometry, radiometrics, deconvolution, inversion

INTRODUCTION

Gamma radiation measurements are made at airborne survey heights and, through suitable processing, are converted to estimates of the count rates due to the radioelements (K, U and Th) at each observation point along the flight lines. The conventional processing of these data would include a correction for deviations in the aircraft height from the nominal survey height. The elemental count rates are then typically converted to estimates of the elemental concentrations on the ground through a simple scaling of the count rates by a "sensitivity factor" that depends only on the energy of the primary radiation and the nominal survey height above ground level (IAEA, 2003). Finally, the elemental count rates are interpolated onto regular grids for imaging and interpretation.

The sensitivity correction is applied on a point-by-point basis, despite the fact that the "field of view" of an airborne detector can be a circle of up to 700 m diameter on the ground, depending on the height of the detector. This results in a "blurring" of spatial detail – anomalies due to sharp discontinuities in radioelement concentrations on the ground are represented in the final airborne data as smooth transitions.

The solution is to invert the airborne data to elemental concentrations on the ground in a rigorous way that accounts for the degradation of the gamma signal with distance from the source, the distribution of radioelement sources in the ground, and the response function of the detector.

Craig et al. (1999) described a method for the deconvolution (or downward continuation) of gridded radioelement data. Billings et al. (2003) extended the work of Craig et al. to incorporate into their model the directional sensitivity of the rectangular detectors in common use today, and the movement of the detector through the air. The deconvolution effectively corrects for both the nominal survey height and the directional sensitivity of the detector with a view to sharpening up the edges of anomalies.

The methods described by Craig et al. and Billings et al. are routinely used for the enhancement of high-quality gridded gamma-ray spectrometric data. However, they are limited in that they are applied to gridded data, and do not account for the earth's topography.

We present a new method for inverting airborne gamma-ray spectrometric line data to a rectangular grid of radioelement concentrations on the ground. The method incorporates the directional sensitivity of rectangular detectors, the errors in the airborne data, and a source (forward) model that incorporates the 3D topographic variations in the survey area. The method is a significant improvement on current methods, and gives superior interpolation between flight lines. It also eliminates terrain effects that would normally remain in the data with the use of conventional gridding methods.

THE INVERSION MODEL

The new method uses a source model comprising vertical rectangular prisms of uniform radioactivity and with the same horizontal dimensions as the required grid cell size. The top of each prism is a plane surface derived from a best-fit plane to the digital elevation model of the earth's surface within each grid cell area.

We start with the two-dimensional gamma-ray response, $f(x,y)$, of an elementary vertical rod terminating at the earth's surface, adapted from the one-dimensional response given by Kogan et al. (1971) and Tammenmaa et al. (1976) as follows:

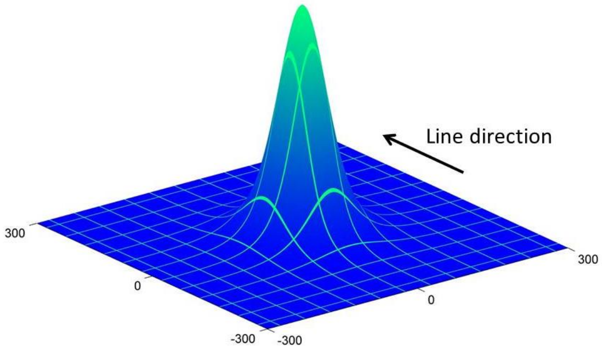

Figure 1. Source-detector response for a slab detector at 60 m height above a vertical rectangular prism source centred at (0,0) with dimensions 50×50 m. The figure is 600 m wide in the x and y dimensions.

$$f(x,y) = \frac{Bh\exp\left(-\mu\sqrt{h^2+r^2}\right)}{(h^2+r^2)^{3/2}}, \quad (1)$$

where $r = \sqrt{x^2+y^2}$, x and y are the lateral distances from the source, μ is the linear attenuation coefficient of gamma-rays in air for a particular energy, h is the height of the detector above the ground, and B is a constant that depends on the radioactivity of the ground, the energy of the radiation, and the sensitivity of the detector. We calculate the 2D response due to a vertical rectangular prism by integrating Equation (1) over the horizontal x and y dimensions of the prism. The response is also corrected for the directional sensitivity of the detector (Grasty, 1975, Tewari and Raghuwanshi, 1987) and the velocity of the detector (Billings and Hovgaard, 1999).

The source-detector response for a 50 m grid cell size (and hence prism size) is shown in Figure 1 for a 60 m flying height. This shows that the response due to a rectangular prism extends well beyond the source, and its effects are measureable on several airborne samples, as well as on at least one adjacent flight line on either side of the cell.

The inversion model is parameterized by considering, for any particular observation point, all prisms that are within the field of view of the detector.

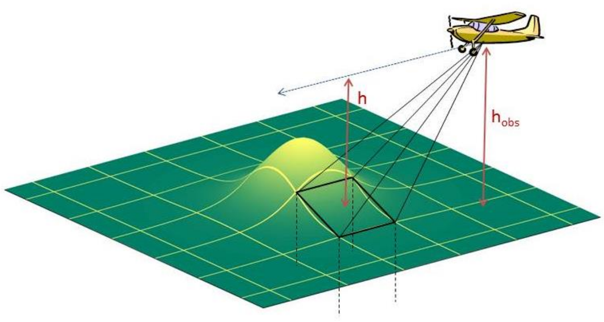

Figure 2. Schematic showing the parameterization of the topography using prism sources with slanted top surfaces relative to the detector.

We incorporate the topography into the model by replacing the top of each prism with a plane surface derived from a best-fit plane to the digital elevation model of the earth's surface within each grid cell area (Figure 2), and appropriately scaling the response (Equation 1) by the ratio of the surface area of the prism top to that of the modelled rectangular prism.

The inversion problem is to estimate the concentrations of M radioactive prisms that best predict the N observed count rates at airborne height. The number of unknowns, M, is typically greater than the number of data, N. We therefore solve by introducing regularization into the inversion through smoothness constraints on the model.

THE INVERSION METHODOLOGY

Using the terminology of Brodie and Sambridge (2006), we wish to minimize an objective function of the form

$$\varphi = \varphi_d + \lambda\varphi_m, \quad (2)$$

where φ_d is a data misfit term, and φ_m is a model roughness term. The regularization factor λ weights the relative importance of the data misfit and model roughness terms. The data misfit is defined as the weighted L_2 norm

$$\phi_d = [\mathbf{Gm} - \mathbf{d}]^{\mathrm{T}} \mathbf{C}_d^{-1} [\mathbf{Gm} - \mathbf{d}], \quad (3)$$

where **d** are the observed data (elemental count rates at airborne heights), and **m** are the unknown model parameters (concentration estimates for each prism). **G** is the sensitivity matrix whose entries, G_{ij} are the contribution of the j^{th} model prism to the i^{th} datum for unit concentration of the radioelement in the prisms. $\mathbf{C}_d$ is the data covariance matrix. For gamma-ray spectrometric data, there is no covariance between datum errors as each measurement occurs independently of every other, so $\mathbf{C}_d$ is a diagonal matrix with elements v, where v is the variance of each datum. We use knowledge of the errors in the raw data (variance = mean count rate for Poisson-distributed counts), and trace the propagation of these errors through the conventional data processing procedures to estimate the errors in the final count rates.

The model roughness term is defined as

$$\varphi_m = \mathbf{m}^{\mathrm{T}}\mathbf{L}^{\mathrm{T}}\mathbf{L}\mathbf{m}, \quad (4)$$

where **L** is the second finite difference operator (..1 -2 1..), which is applied in both E-W and N-S directions.

We wish to find the model parameters where

$$\frac{\mathrm{d}\phi}{\mathrm{d}\mathbf{m}} = 0. \quad (5)$$

It can be shown that this is where

$$(\mathbf{G}^{\mathrm{T}}\mathbf{C}_d^{-1}\mathbf{G} + \lambda\mathbf{L}^{\mathrm{T}}\mathbf{L})\mathbf{m} = \mathbf{G}^{\mathrm{T}}\mathbf{C}_d^{-1}\mathbf{d}. \quad (6)$$

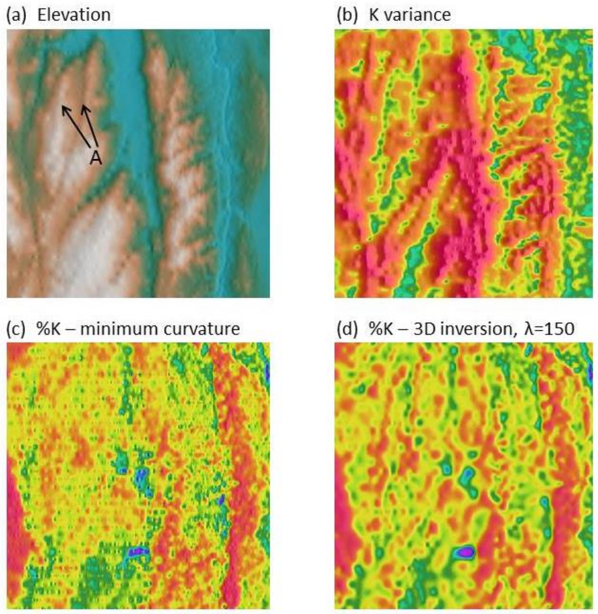

Figure 3. Pseudo-colour images from the SE Lachlan survey area: (a) digital elevation model, (b) K concentration variance, (c) K concentration (%) gridded using minimum curvature, and (d) K concentration gridded using 3D inversion. The images are centred on (149:04:39.99 E, 35:40:25.74 S) and are about 15 km wide. (Data courtesy of the Geological Survey of NSW)

To invert our airborne data to a grid of elemental concentrations we solve the linear system (Equation 6) using the preconditioned conjugate gradient method implemented via the open-source PETSc code (Balay et al., 2014).

APPLICATION TO AIRBORNE DATA

Figure 3 shows an example from the South-East Lachlan survey, New South Wales. The survey was flown at 60 m height over an area that is quite mountainous in parts. Figure 3a shows the digital elevation model over part of the survey area, which shows elevation changes of up to 900 m.

The inability of the fixed-wing aircraft to fly a drape surface has resulted in large deviations from the nominal survey height, which then propagates as errors into the final radioelement estimates. In some places the aircraft was more than 400 m above ground level. The K concentration variances are shown in Figure 3b and vary in places by more than a factor of 20.

The minimum curvature grid is shown in Figure 3c and the 3D inversion equivalent in Figure 3d. The 3D inversion grid is a significant improvement on the minimum curvature grid as follows:

- The 3D grid shows better continuity of anomalies and interpolation between flights. The positions of the flight lines are evident in the minimum curvature grid as E-W lines of high-frequency speckle corresponding to the original observation points.

- The 3D gridding has a natural tendency to smooth the grid in areas where the error variances are large. This is a logical consequence of the data being given less weight, relative to the smoothing term in Equation 6, as the errors in the data increase. This is an improvement on the minimum curvature grid which tends to grid noise in these areas.

- The incorporation of the topography into the inversion can be clearly seen at A, and elsewhere, in Figure 3. In the deeply-weathered Australian environment the tops of mountains and ridges, which are actively eroding and thus exposing fresh rocks and soils, often show higher K concentrations than other more deeply-weathered parts of the landscape (Wilford, 1997). As the aircraft passes over a ridge, many of the sources within the field of view are further from the detector than is the case for a flat earth. In these circumstances conventional methods underestimate radioelement concentrations as the data are corrected for the height of the ground directly beneath the aircraft assuming flat-earth geometry. The opposite is true over valleys, where conventional methods tend to overestimate the concentrations directly beneath the aircraft. This effect is evident along the ridges at A in Figure 3 – the 3D inversion is correctly identifying the increase in K concentration whereas the conventional processing is not.

CONCLUSIONS

The new method is more than just a gridding technique. It is a physics-based method for the rigorous inversion of airborne gamma-ray spectrometric line data to elemental concentrations on the ground. The method incorporates the height of the aircraft, the 3D terrain within the field of view of the spectrometer, and the directional sensitivity of the rectangular detectors currently used in airborne surveying. The resulting grids of elemental concentrations are a significant improvement on those derived using conventional methods. The 3D inversion gives better interpolation between flight lines, and incorporates our knowledge of errors in the data into the final grid. It also eliminates terrain effects that would normally remain in the data with the use of conventional processing methods.

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