

# INNOVATIVE DATA PROCESSING METHODS FOR GRADIENT AIRBORNE GEOPHYSICAL DATASETS

**Desmond J FitzGerald; Horst Holstein.**

Intrepid Geophysics, Melbourne, Australia;  
University of Wales Aberystwyth, Wales, UK.

## ABSTRACT

Intrepid Geophysics has adapted its existing software to include a new series of objects that can be used when processing gradient datasets. These have the purpose of hiding the details (abstraction) of exactly what components of a field have been observed in a survey dataset. This avoids the problem of doing a general rewrite of processing software from the ground up for each special case. Historically, codes have mostly been written to filter, level and grid “scalar” line data (e.g. Total Magnetic Intensity), so Intrepid Geophysics latest software update is a dramatic shift to the world of vectors & tensors.

The new family of classes in this adapted software is designed to honour all the commonly available airborne geophysical observation packages. Specifically, for magnetic gradiometry systems, the magnetic intensity plus:

1. vertical gradient only,
2. transverse gradient (wing tip sensors),
3. transverse & longitudinal gradient (wing tip & tail stinger),
4. all gradients (full tri-axial system),
5. all components of a field,
6. full 2<sup>nd</sup> order tensor gradients.

For moving platform gravity, the vertical component (if available) plus:

1. vertical component plus motion monitors (L&R / ZLS),
2. 2 horizontal curvature tensor (Falcon system),
3. 3 gravity components (Sander),
4. full 2<sup>nd</sup> tensor gradients (Bell),

With this approach, each derived class is delegated the task of enforcing any appropriate invariant relationships e.g. tensor symmetry, trace invariance, rotational invariance, boost symmetry, etc. This innate behaviour can be relied upon to carry through in any process involving a manipulation of a measurement with another reading. This greatly assists the development of algorithms that work with all the various systems in a physically consistent way.

The basic core functions, namely of database support, vector statistics, visualization mimic of each sample, gridding, filtering and levelling have proven to be viable for large surveys.

## INTRODUCTION

Data processing of geophysical data has historically been based on scalar processes. Intrepid Geophysics has adapted its software to now take into account vectors and tensors in order to improve and enhance its data processing techniques. This modified software uses vectors and tensors in an object-oriented design where the details are mostly hidden to the application software and only exposed where required.

### Test Data Sources

Survey areas used to test the new software functions were the 2002 African airborne gravity dataset collected using the Bell system; the 2003 Timmins Airborne Gravity dataset collected using the Sander system and the 2004 Baker airborne magnetic gradient dataset collected by Firefly for Tanqueray Resources.

The list of core functionality for a vector / tensor processing system includes:

- database support for new data types
- mimic graphics
- group statistics
- interpolation
- filtering

- levelling
- residual anomaly calculation
- interpretation

## OBJECT ORIENTED DESIGN ELEMENTS

The initial object oriented design used by the adapted software to hide details of the actual data collected is shown below in Figure 1.

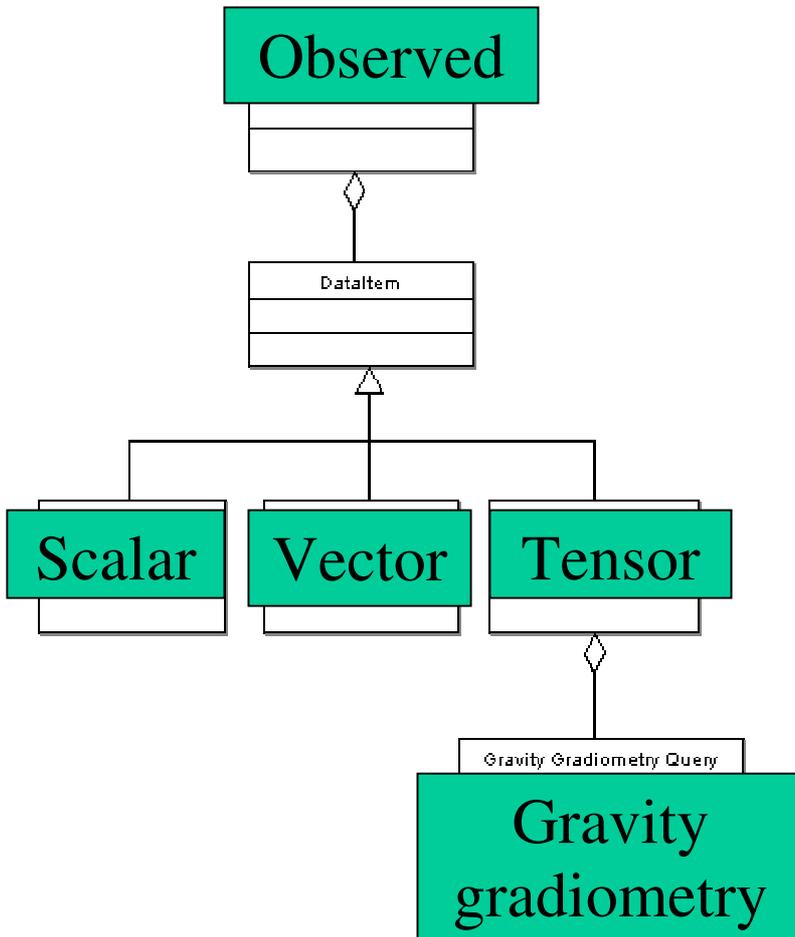


Figure 1. Object oriented design used by the adapted software to hide details of the actual data collected. This design example shows Gravity gradiometry as the base class.

Refinement of this design, once the base class (eg. Gravity gradiometry) is established, progresses with minimum impact on applications and other libraries. Five variations on the vector class are immediately required for service, namely Magnetic and Gravity gradients, Magnetic and Gravity Components and a directional cosine vector.

## VISUALIZATION STRATEGIES

Patterns in measured field data and their gradients are more easily grasped if graphical representations or mimics are used.

### Tensor

A traditional means of understanding the relationship between tensor components is accomplished using a Mohr (1900) circle diagram, as shown in Figure 2. This graphical technique was developed by engineers as a means of solving for the principal components before computers were invented. In the context of a processing system, a spreadsheet editor has been adapted to show Mohr's circles for an observed series of tensor readings.

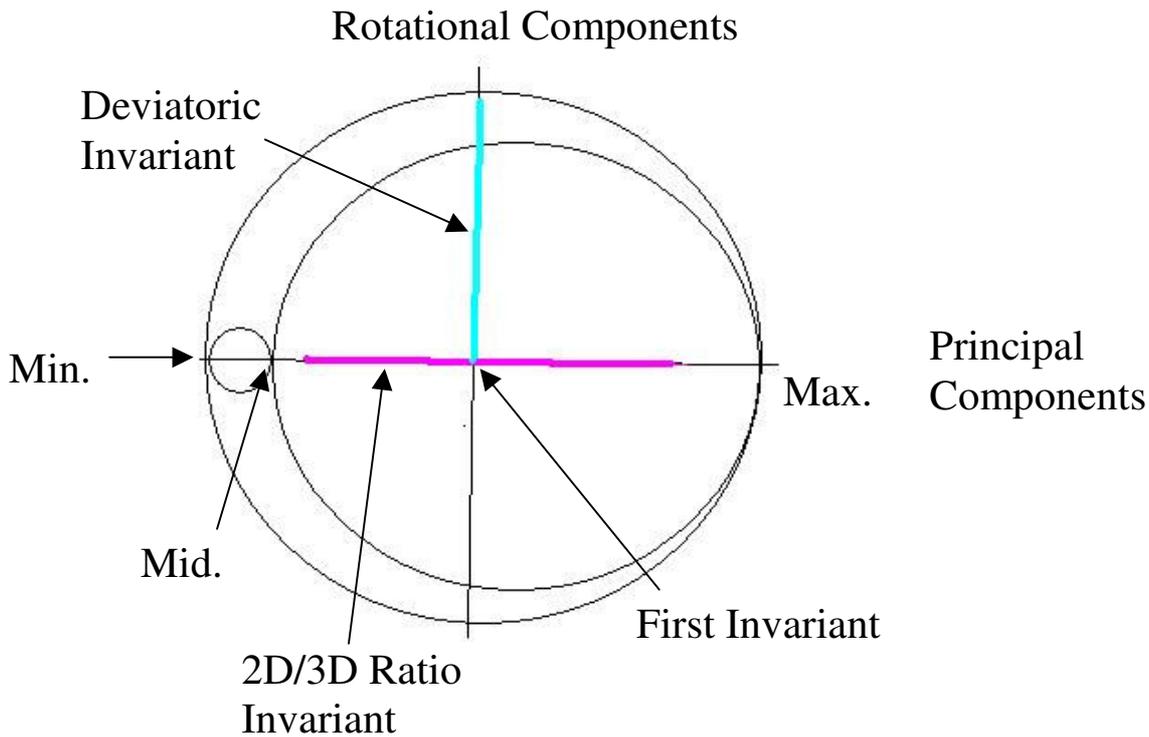


Figure 2. Tensor mimic Mohr (1900) circle diagram showing the relationship between tensor components.

In the case of gravity and magnetic gradiometry, the First Invariant of the tensor is supposed to be zero, and so a vertical axis is shown at this point. It would be expected that the raw gravity gradients radius be approximately 308.6 Eotvos, the normal Free Air vertical gravity gradient term. The tensors are symmetric, so only the top half needs be shown. The horizontal axis represents the normal or principal components and the vertical axis represents the rotational gradients.

Each tensor has its principal components solved and used as a basis for drawing each circle scaled to the maximum difference in components for the current group.

In addition, the spatial component of the field tensor can be expressed most economically in its quaternion representation. This is a 4-dimension space that allows the successive angular changes to be shown in the standard 3 views of plan, long and cross.

Figure 3 shows a snap of the Intrepid Geophysics spreadsheet tool displaying each individual tensor before and after two filtering processes. This data is from a 2002 African airborne gravity dataset collected using the Bell system.

The Grav\_Lev channel is as delivered from Bell, the Grav\_2k2d channel is the tensor filtered by a low pass on each individual component separately and finally, the Grav\_RC is an RC filter applied to the tensor as a whole. Subtle changes are seen here. These are more obvious as one scrolls through the records.

One possible critical approach is to examine the preserved ratios of the invariants using the Mohr circles.

If the tensor signal has been compromised either during the acquisition, or by processing, it is immediately obvious when displaying the data in a Mohr circle – the characteristic relationships between the circles are not present and all you see are the axes.

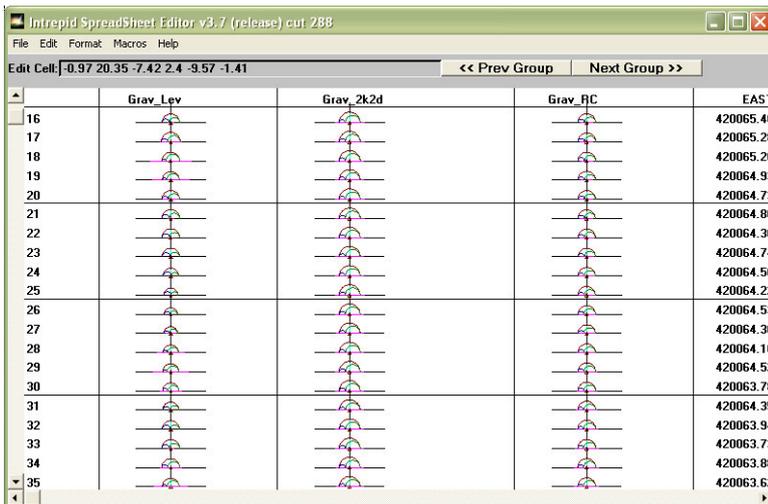


Figure 3. 2002 African tensor gradients mimic display, showing original and filtered gravity data.

### Vector

For gravity components, as measured by a system such as Sander, the predominant signal is the traditional vertical component. The maximum horizontal component swings around all points of the compass, reflecting the lower signal to noise ratio as much as the density variations. The graphical mimic shown in Figure 4 is proposed for this case.

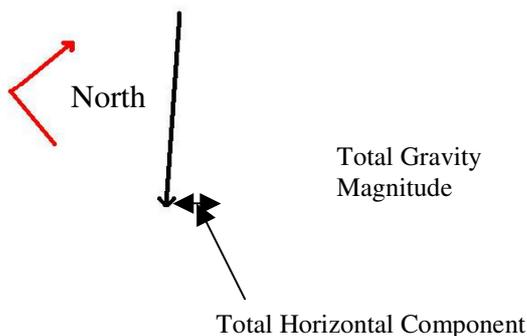


Figure 4. Gravity Components Mimic Display

Figure 5 is a sample of the 2003 Timmins dataset from Sander showing the gravity component displayed in a mimic of the above form.

	G_acc	date	z	Height	altimet
198		17/05/2003 14:18:14	460.15	496.43	216.0
199		17/05/2003 14:18:14	459.98	496.26	215.0
200		17/05/2003 14:18:15	459.86	496.14	215.0
201		17/05/2003 14:18:15	459.84	496.12	215.0
202		17/05/2003 14:18:16	459.94	496.21	Nu.
203		17/05/2003 14:18:16	460.14	496.41	Nu.
204		17/05/2003 14:18:17	460.39	496.66	Nu.
205		17/05/2003 14:18:17	460.61	496.88	Nu.
206		17/05/2003 14:18:18	460.75	497.02	Nu.
207		17/05/2003 14:18:18	460.76	497.03	Nu.
208		17/05/2003 14:18:19	460.62	496.89	Nu.
209		17/05/2003 14:18:19	460.36	496.63	Nu.
210		17/05/2003 14:18:20	460.03	496.30	Nu.
211		17/05/2003 14:18:20	459.69	495.96	Nu.
212		17/05/2003 14:18:21	459.40	495.67	Nu.
213		17/05/2003 14:18:21	459.17	495.44	Nu.
214		17/05/2003 14:18:22	459.00	495.27	Nu.
215		17/05/2003 14:18:22	458.84	495.11	Nu.
216		17/05/2003 14:18:23	458.65	494.92	Nu.
217		17/05/2003 14:18:23	458.38	494.65	Nu.

Figure 5. 2003 Timmins gravity dataset displayed using an Intrepid Geophysics mimic.

A different mimic for magnetic gradients has been developed, but it is not shown here.

## STATISTICAL QUANTITIES

Another immediate challenge is to create summary statistics for these data types. There is a very large and well-established set of methods for directional field vector data that is used in palaeomagnetism. Fisher (1953) & McFadden (1980) suggested that the distribution of vectors on a unit circle is analogous to a normal distribution.

For tensor gradient data, the principal components are pressed into service to act as Maximum, Minimum and Mean analogues. The 2003 Timmins dataset and 2002 African dataset are used to illustrate this enhanced statistical reporting, as shown in Figure 6 and Table 1.

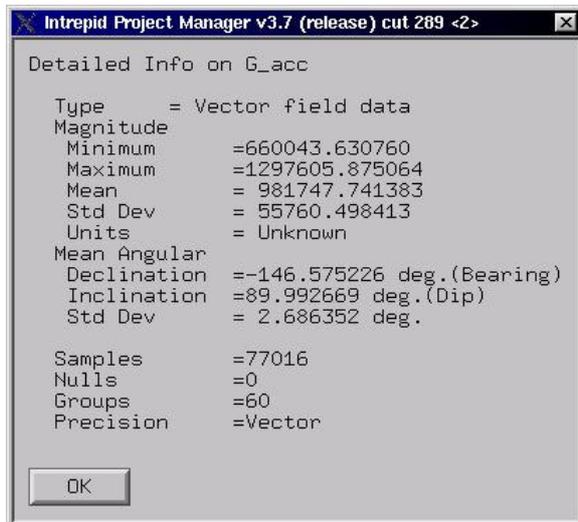


Figure 6. 2003 Timmins gravity dataset statistics calculated using the Fisher (1953) & McFadden (1980) technique.

Magnitude	1724	-148	-1537
Declination	17	-9	53
Inclination	73	13	-13
Std.Deviation	3.5	12	3.5

Maximum                      Middle                      Minimum

Table 1. 2002 African gravity gradiometry dataset - tensor statistics (eigenvectors).

## GRAVITY GRADIENT CORRECTIONS

### Free Air

Taking a similar approach to the scalar field, a theoretical gravity gradiometer component should be subtracted from the observed gravity tensor to calculate the Free Air tensor.

The theoretical First Order tensor correction at the any elevation can be derived. For sea level it is approximately

$$154.3 \begin{bmatrix} 1 & & \\ & 1 & \\ & & -2 \end{bmatrix} \text{ Eotvos}$$

assuming an upward vertical direction for the third Cartesian measurement axis

### Terrain Correction

This is a very compute intensive operation. The availability of gravity gradient formulae (Horst 2004) as opposed to local differencing of fields, makes this operation more efficient.

## GRIDDING

### Interpolation Strategies

Both real data and synthetic model data that are representative of each observed data variation is used. Typical interpolation schemes in potential field algorithms are:

1. Akima Spline (use observed gradient transformed to be along direction of spline),
2. Minimum Curvature (Briggs, 1974 and O'Connell et. al. 2005)
3. Nearest Neighbours (blend gradient contribution with field estimate).

The key question is how gradient information can be used to create a superior representation of the field during interpolation. Each multiple, addition and division involved is examined to see how this should be implemented when Vector or Tensor components or gradients are involved. Having looked at each case, it proved possible to define “appropriate overloaded operator rules” for each case, and so in the pre-existing application codes, there was very little change evident.

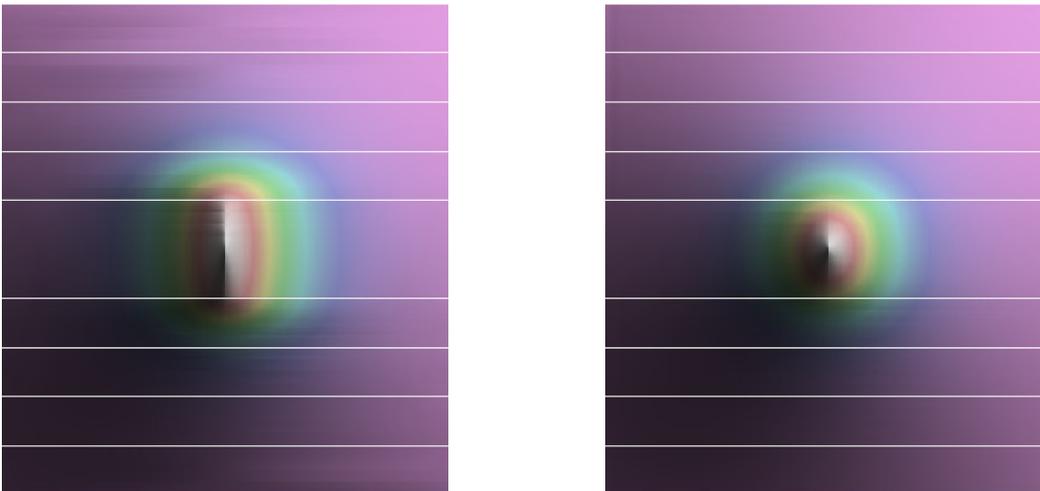
A typical change might be that a weighting factor is forced to be a second term in a multiplication, instead of being either first or second. For example:

$$\text{oldZ} = \text{wt1} * z1 + \text{wt2} * z2$$

is changed to be:

$$\text{newZ} = z1 * \text{wt1} + z2 * \text{wt2}$$

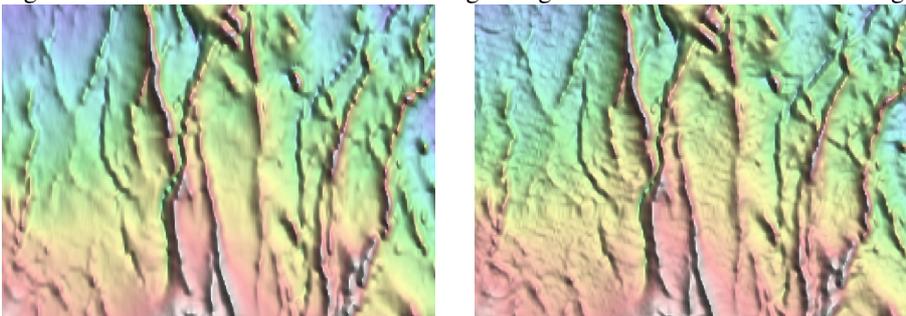
This simple change helps a compiler recognize we are dealing with these smart observed objects, and chooses the correct way to calculate the overall result.



Figures 7a and 7b. Sample magnetic data gridded with a missing observation line – before and after gradient enhancement.

Figure 7a shows an image of standard gridding, while Figure 7b shows gradient enhanced gridding. The missing observation line is used to stress test the algorithm.

Figures 8a and 8b show the 2004 Baker magnetic gradient dataset before and after gradient enhancement.



Figures 8a and 8b. 2004 Baker magnetic gradient dataset before and after gradient enhancement.

Note, the higher gradient portion of the signal helps define the dykes into tighter-thinner bodies. The low gradient (noisier) portion introduces a “ripple” into the background.

## TENSOR INTERPOLATION

The recommended method to interpolate between 2 observed tensors involves more than just linear interpolation of each component. This process, if pursued, will rapidly compromise the observed signal. There are not only magnitude changes, but also angular variations. It is recommended that the eigen values together with the associated quaternion of the tensor be used for the filtering and interpolation processes as this will more correctly handle both the magnitudes and angular variations.

This idea is pursued in the appendix.

## LEVELLING

Levelling usually involves one or more of the processes shown in Table 2. The applicability and implementation status in Intrepid Geophysics software is presented.

Correction	X-overs	MAG GRADIENT	GRAVITY TENSOR
GRF		Not needed	N.A.
DIURNAL		?	N.A.
HEADING		Available	Available
PARALLAX		Not done	Not done
POLYNOMIAL	√	Not done	Not done
Network Adjust	√	Available	Available
X Y Position	√	Not done	
Observation Height	√	Unfinished	Signal / Noise issues

Table 2. The various processes involved in levelling magnetic and gravity data and their current applicability and implementation status in Intrepid Geophysics software.

### Magnetic Gradient Field Data

Misclosure at a crossover point for a field becomes the vector difference. Some questions are:

Does the Observation Instrumentation’s calibration drift in time? If so, how? Diurnal corrections would also be a vector operation, but so far, no one has base stations measuring xyz gradients.

### Tensor Gradients

Misclosure tensor has a much higher signal/noise ratio than the signal down each profile. This immediately is obvious in the direction statistics for their misclosures, as shown in Table 3, compared to the previous statistics in Table 1. For the eigenvector of the maximum eigenvalue, the angular standard deviation goes from 3.0 to 56 degrees.

Magnitude Eotvos	489	3.41	-490
Declination degrees	68	27	31
Inclination degrees	55	13	76
Std.Deviation	56	45	60

Maximum                      Middle                      Minimum

Table 3. 2002 African gravity gradiometry dataset misclosure tensor statistics based on 1200 crossovers.

Figure 9 displays acquisition and tie line crossovers from Intrepid Geophysics levelling tool for the 2002 African tensor gravity gradiometry dataset.

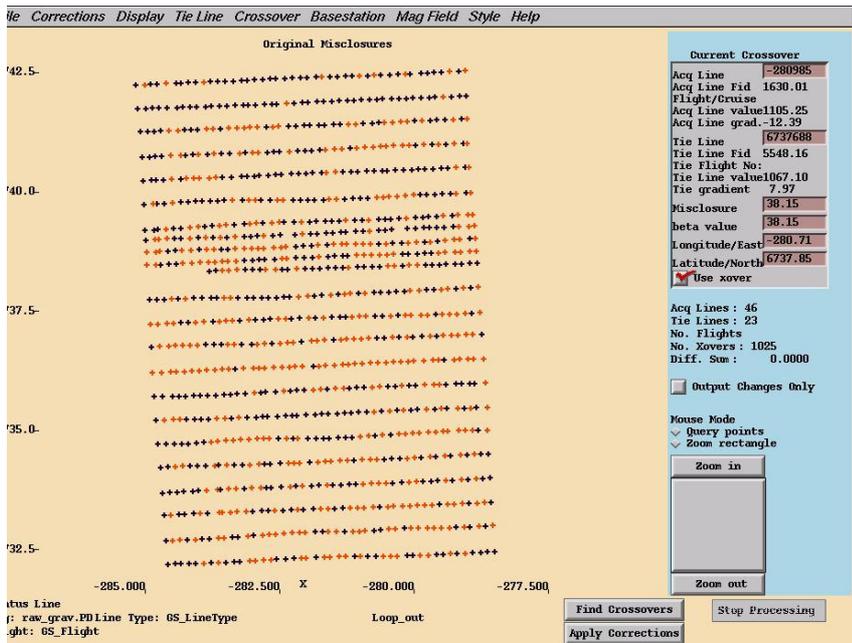


Figure 9. Acquisition and tie line crossovers from Intrepid Geophysics levelling tool for the 2002 African tensor gravity gradiometry dataset.

## FILTERING

The signal to noise ratio of measured gradients can be low. Consequently, innovative noise reduction using IIR (recursive techniques) are required. Also, IIR filters are needed to integrate the measured gradient and Ticks formula is under trial.

A spatial convolution, using an odd operational length (eg. 5), is used in the filtering of vector and tensor components. Filters currently tested include a moving average, median, LaCoste RC and Laplace curvature (damped).

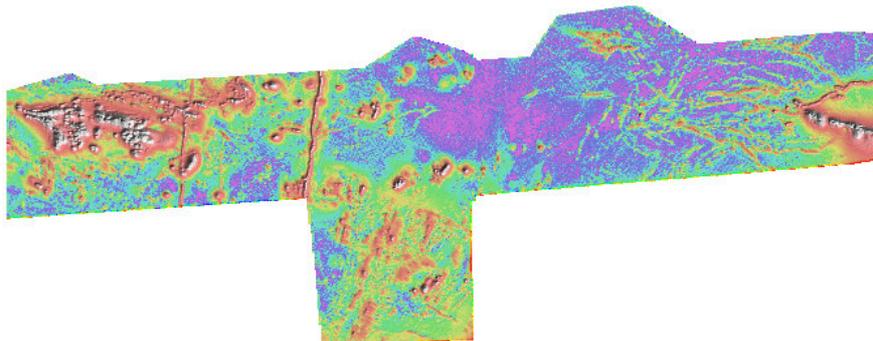
An attempt is made to honour the invariants of a tensor while a noise or spike-filtering operation is performed. The LaCoste RC filter code is used as a test on observed Bell tensor data to explore the possibilities. This filter is recursive, and dampens the noise.

The Laplace filter is adapted from the L. Lacoste filter for curvature calculation and filtering. It uses a stack of original and transformed or filtered observed values and uses the time in seconds ( sampling time say 1 sec or 10 sec). It is similar in operation to the RC or Kalman filter but works on the whole tensor/vector.

## Interpretation Products

Interpretation products can now be produced directly from some of these new software options. Data gridded with gradient enhancement can be used to improve and define features for easier interpretation. The case for the complex analytic signal from a triaxial gradient system is also compelling. Figure 10 displays the magnetic data from the 2004 Baker dataset showing a calculated analytic signal of the data. It is arguable that this can be done straight from observed gradients or from FFT transforms as below.

Figure 10. 2004 Baker magnetic gradient dataset showing an analytic signal of the data (Band 0 : Amplitude, Band 1 : Tilt Angle).



## **INVERSION**

The GeoModeller Inversion capability includes support for a full geologically constrained gravity tensor gradients. An adoption of Holstein (2004) is in progress to allow for variable depth discretization into triangles of the starting geological model and then modelling of the gravitational and magnetic model.

## **CURRENT STATUS**

The Intrepid Geophysics software processes that have progressively been updated to include support for this smart observed data object are:

1. Gridding
2. Profile Editing of the complete Tensor for damping Noise
3. Loop levelling
4. Spreadsheet editor – mimic displays
5. Project manager support
6. Import into new persistent database types
7. Statistical improvements
8. Free Air corrections
9. Terrain corrections
10. Forward and Inverse Geology constrained 3D modelling.

It can be said that there is a very large increase in the use of computer Heap space when these methods are used, but this challenge is easily met by newer generation desktop computing systems.

## **CONCLUSIONS**

Smarter computational support for new generation geophysical datasets is here to stay.

A greater use of Object Oriented methods can contribute to controlling and right sizing the complexity of each process.

Processing the “OBSERVED” package as an object can be achieved:

- Hides details from processes that do not need to know
- Presents field physics issues naturally

Most of the needed technology already exists in closely related geoscience disciplines.

Instrumentation engineers should be encouraged to gather still more real-time characteristics and to report them, as there is little excuse for not being able to extract more value from this data post-mission.

Aircraft compensation should be re-thought to take the field nature of the signal and its gradients into account.

## **REFERENCES**

Briggs, I.C., Machine Contouring Using Minimum Curvature, *Geophysics*, Vol. 39, NO. 1 (February 1974), p. 39-48, 3 Figs, 3 Tables.

Fisher, R.A. (1953). Dispersion on a sphere. *Proc. Roy. Soc. London*, A217, 295-305.

Holstein, H., Reid, A.B. and Sherratt, E.M. (2004). Gravimagnetic field tensor gradiometry formulas for uniform polyhedra. Presented at the 74<sup>th</sup> Annual Meeting of the SEG, 10-15 October, 2004, Denver, Colorado.

McFadden, P. (1980). The best estimate of Fisher's precision parameter  $J$ , *Geophy. J.R. astr. Soc.* Vol. 60, pp 397-407.

Mohr, O. (1900). Welche Umstände bedingen die Elastizitätsgrenze und den Bruch eines Materials? *Z. Ver. Dt. Ing.*, 44, 1524-30; 1572-77.

O'Connell, Michael, Smith, R.S. and Vallee, M.A. (2005). Gridding aeromagnetic data using longitudinal and transverse horizontal gradients with minimum curvature operation, *Leading Edge*, Vol 24, No. 2, pg 142.

## APPENDIX – EIGEN-REPRESENTATION OF GRAVIMAGNETIC FIELD GRADIENT TENSORS FOR INTERPOLATION AND FILTERING PURPOSES

Gravity and magnetic field gradient tensors are known to be symmetric and of zero trace (sum of diagonal components). This admits 5 degrees of freedom among the 9 components present in the full 3-dimensional tensor of rank 2. Interpolation and filtering processes must honour these dependencies.

Analogously to vectors, tensor components depend on the choice of measurement coordinate system. It is, however, possible to describe the tensor in terms of disjoint structural and orientational properties. The structural properties are *independent* of the choice of coordinate system, while the orientational properties depend *only* on the choice of coordinate system. This gives rise to the possibility of interpolating these two properties separately, in a manner that allows reconstruction of an interpolated tensor at an “in between” field measurement point. These remarks similarly extend to filtering.

Let  $\mathbf{T}$  be the 3x3 matrix associated with a field gradient tensor in a given Cartesian coordinate system. The matrix  $\mathbf{T}$  will be symmetric and satisfy  $\text{trace}(\mathbf{T})=0$ . For such a matrix, there exists a 3x3 rotation matrix  $\mathbf{R}$  satisfying

$$\mathbf{R}^T \mathbf{T} \mathbf{R} = \mathbf{\Lambda}, \quad (1)$$

where  $\mathbf{\Lambda}$  is a 3x3 diagonal matrix containing the eigenvalues of  $\mathbf{T}$ , all real. This is a result from standard eigensystem construction. Moreover, the three eigenvalues are real-valued, with preserved trace,

$$\text{trace}(\mathbf{T}) = \text{trace}(\mathbf{\Lambda}), \quad (2)$$

and the columns of  $\mathbf{R}$  form three orthonormal vectors that define the unit axes of an orthogonal coordinate system. In this coordinate system, the tensor has the diagonal matrix representation  $\mathbf{\Lambda}$ . Standard mathematic software can determine matrices  $\mathbf{R}$  and  $\mathbf{\Lambda}$ .

The structural information of the target that gives rise to the field gradient anomaly is contained in the matrix  $\mathbf{\Lambda}$ . This information can be conveniently interpolated or filtered in terms of operations on each of the three diagonal components, subject to the trace condition.

The orientational information is contained in the rotation matrix  $\mathbf{R}$ . It is known from the theory of rotation operators that matrix  $\mathbf{R}$  has a unit 4-vector representation, (strictly speaking, a unit quaternion). Thus, the sequence of observed field gradient tensors can be associated with a path traced out on the surface of a unit 4-sphere. Interpolation is to be carried out in this manifold, to yield a new unit 4-vector interpolant. This interpolant allows the corresponding rotation matrix to be reconstructed. Effectively, we have an interpolation process for rotation matrices that yields only rotation matrices. Similar remarks hold for the filtering operation.

The decomposition into structural and rotational parts yields 2 plus 3 independent quantities respectively, conforming to the 5 degrees of freedom of the original formulation. The decomposed form, however, allows interpolation or filtering processes to admit only consistent matrix representations of the underlying tensors with regard to their structural and rotational field gradient information content.