Lithologically constrained inversion of magnetic and gravity data sets

R. Lane
Geoscience Australia, Canberra
richard.lane@ga.gov.au
D. FitzGerald
Intrepid Geophysics, Melbourne
des@intrepid-geophysics.com
A. Guillen
Intrepid Geophysics, Melbourne
R. Seikel
Intrepid Geophysics, Melbourne
P. McInerney
Intrepid Geophysics, Melbourne

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1 Abstract

A gravity and magnetic potential field litho-inversion scheme that we have used to evaluate and refine 3D geological maps is briefly described. When producing such maps, there are only a limited number of direct geological observations to constrain the distribution of geological units in the volume of interest. A very large number of geological models that satisfy these *a priori* geological constraints could be generated. Using a Bayesian approach to inversion, we reduce the level of uncertainty by identifying a subset of the possible models that can reproduce a set of geophysical observations in addition to the *a priori* geological constraints.

A 3D ‘reference’ geological model that contains the *a priori* geological knowledge is built. Using supplied estimates of the physical properties associated with each of the geological units, property models are generated that are directly related to this geological model. The gravity and magnetic fields of these property models can thus be calculated. A likelihood is derived for this combination of geological model, property models and geophysical observations by comparing the calculated response with the geophysical observations.

A large sample of the possible geological models is generated in an iterative fashion by making incremental changes to the lithological regions and/or the properties. A likelihood is derived for each new combination. We gain geological insight from the inversion by isolating and statistically analysing the models with high likelihood with respect to the potential field data. As a consequence of generating many different geological models in the course of the inversion, we can begin to explore the geological uncertainty in the inversion results. An overview of a case study for the method is presented for the Bet Bet region in central Victoria, Australia.

2 Introduction

With increasing demand for 3D geological models, there is a need to produce models more efficiently, and, coupled with this, a need to quantify the reliability of the models. To build and re-build 3D geological models rapidly, an implicit mathematical functions method is used (Lajaunie et al. 1997; Calcagno et al. 2006). This method allows the model to be constructed directly from a range of geological observations, which also means that the model can be easily revised from time to time as new data become available.

The validity of the 3D geological model can be estimated statistically by inverting complementary geophysical datasets. Gravity and magnetic potential field data are useful in this context because:
(1) the data are a function of the 3D distribution of a source,
(2) the response of the 3D source distribution can be calculated, and
(3) the source distribution shows a degree of correlation with geological litho-regions.

Unfortunately, inversion of these data will not return a unique property source geometry solution, nor is it likely in practice that the source geometry will be perfectly correlated with the litho-regions. Despite these limitations, we can still obtain useful information by using a Bayesian approach to inversion. This approach begins with the recognition that direct observational knowledge of 3D geology is imperfect. For any set of geological observations and assumptions (i.e., the *a priori* geological information), there is an infinite number of geological models that could be generated, and perhaps a large but finite number of “significantly different” models. The number of permissible models can be refined by relating each geological model to a set of independent geophysical observations through observations and/or assumptions about the physical properties associated with each of the geological units (i.e., the *a priori* property information) and the laws of physics, which we assume are correct. We can quantify the likelihood of a geological model (i.e., the *a posteriori* probability) by comparing the calculated and observed geophysical data, with the likelihood increasing as the misfit decreases. If we sample the range of possible geological models, we will be able to identify those that have significant likelihood, and hence have used the geophysical data to gain insight into the likely geological architecture of the model region.

To reduce the non-uniqueness and to increase the geological information that can be recovered from gravity and magnetic data, any number of scalar, vector or tensor components of each potential field can be included in a simultaneous inversion.

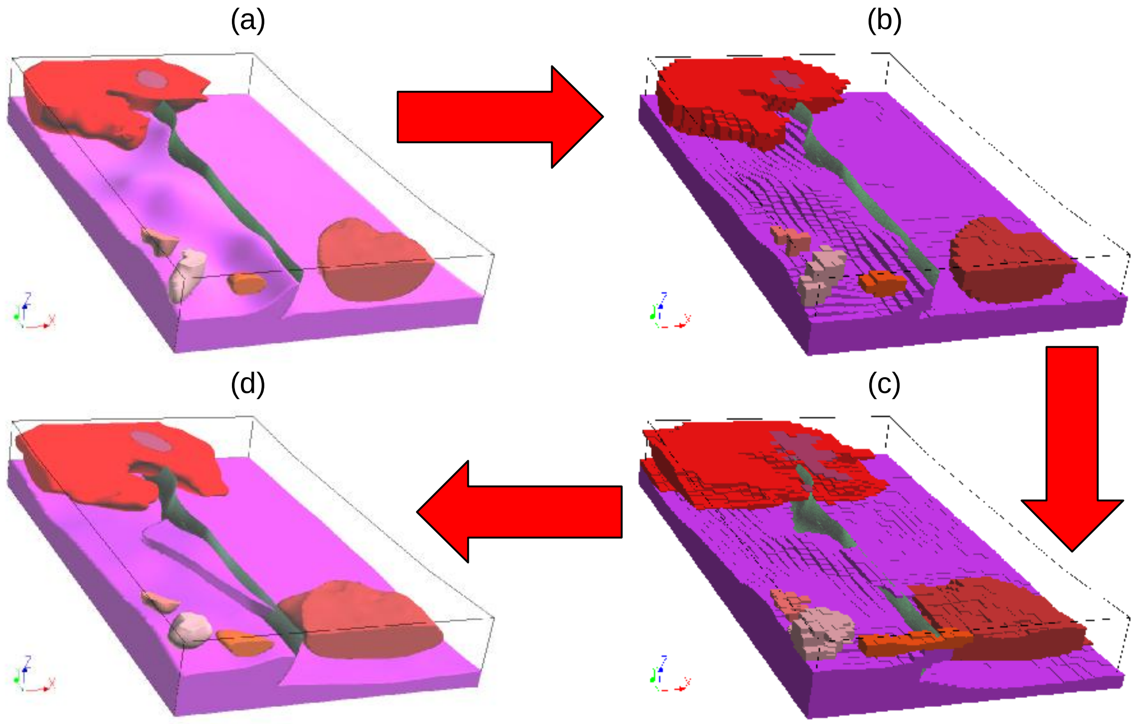

Fig. 1. Summary of the Bet Bet inversion example. Model dimensions are 33 km E/W, 57 km N/S and 7.7 km vertically. (a) Initial geological model with meta-sedimentary units removed. (b) Initial model in voxel format ready for inversion. (c) ‘Most probable’ composite model from the inversion study. (d) Improved geological model after the inversion study.

3 Outline of the inversion algorithm

Although every inversion has a unique workflow, the inversion procedure can be generalised into a sequence of 4 phases (Table 1).

Table 1. Outline of the steps involved in the inversion algorithm.

Phase 1 - Initialisation Phase
(1a) Build the *a priori* geological model (1b) Define the *a priori* physical property laws (1c) Specify any *a priori* geological constraints (e.g., 'shape' and 'commonality' parameters) (1d) Discretise the model to a voxel-based lithologic model (1e) Specify *a priori* 'fixed' voxels (1f) Make a list of the geological-boundary or frontier cells (1g) Compute a unit-response kernel for the gravity and/or magnetic field and/or their tensor components for each voxel at each observation location (1h) Initialise the models of density, induced magnetisation, and remanence, and initialise any values required for the geological tests (1i) Compute the geophysical effects of the model
Phase 2 – Forward Model Checks
(2a) Confirm that the responses from Step 1i reproduce the first order features of the observed geophysical data
Phase 3 – Iteration Phase
(3a) Select a voxel at random and postulate a change to the model (3b) Assess the geological acceptability of the changed model (3c) If geologically accepted, compute the geophysical responses of the changed model (3d) Using the geophysics misfits, compute the likelihoods of the changed model and decide whether to accept the postulated change to the model or revert to the previous model
Phase 4 – Analysis Phase
(4a) Analyse the ensemble of models and generate various statistics (4b) Visualise inversion products in the context of the reference geological model, noting concordant and discordant regions (4c) Modify the reference geological model in the discordant regions if a single representative geological model is required or communicate the full Bayesian outcome that includes an uncertainty assessment

3.1 Phase 1 – Initialisation Phase

A 3D geological model describing the 3D geometry of geological units is built from a combination of geological observations and inferences. The model is discretised into a 3D matrix of voxels to produce a reference lithological model. There is considerable ambiguity in the way a 3D geological model is constructed from sparse observations. To decide if we have chosen a viable configuration, we evaluate the model with respect to independent geophysical observations through Bayesian inversion.

3.2 Phase 2 – Forward Model Checks

The forward model check that is performed in Phase 2 is subjective at this stage. It is carried out to ensure that the basic architecture of the geological model is consistent with the potential field data, and by doing so, ensure that the inversion procedure will be successful in a reasonable period of time. If there is very little correspondence between the major features of the observed and calculated geophysical data, the user needs to return to Phase 1 and re-consider the *a priori* geological model and the *a priori* physical property laws (Steps 1a and 1b).

3.3 Phase 3 – Iteration Phase

A Bayesian inversion approach requires many alternate geological configurations to be generated and examined for consistency with the observed gravity and magnetic data. We limit the variations applied to the geological model by requiring each model to have certain similarities with the reference model. These similarities are expressed as geological constraints, which are a reflection of the geological observations and the geological assumptions that are believed to be true (at least for this exercise). The statistical sampling procedure that is used ensures that we concentrate sampling of the geological models on those that reproduce the supplied gravity or magnetic data to within a desired tolerance level. The adopted strategy is thus a 'guided random walk' around the space of possible models for a given set of *a priori* information. This approach is described mathematically by Mosegaard and Tarantola (1995) and has been applied to potential field inversion in 2D by Bosch et al. (2001).

An iterative procedure is used to generate many postulated geological models. At each iteration, we make one of two possible changes (Step 3a).

(1) The physical property values (density and/or magnetic susceptibility and/or remnant magnetisation) for a randomly selected voxel anywhere in the model may be modified. The new physical property values are obtained through random selection from the probability function of the relevant physical property distributions supplied by the user for the lithological unit assigned to this particular voxel.

(2) Alternately, the lithology of a voxel that lies on the interface between two or more units may be modified. The postulated new lithology is obtained by random selection from the set of lithologies assigned to the voxels in the immediate neighbourhood of the target voxel. Since a change of lithology constitutes a small change to geological boundaries of the litho-model, such a postulated revision of the geological voxel model is first examined for consistency with the *a priori* geological constraints. One or more geological constraints or tests may be applied (Step 3b), including:

– ***Fixed cells***, where the geological assignment in certain cells is kept fixed at the reference value throughout the procedure.
– ***Shape ratio***, where the 'shape' of a unit, based on the ratio of that unit's surface area to volume, is maintained statistically between limits.
– ***Commonality***, where the degree of overlap in the distribution of each unit relative to the configuration in the reference lithology model is controlled statistically.

The specification of 'shape' and 'commonality' constraints is guided by our level of confidence in the initial distribution of each unit in the reference model.

At Step 3b, the postulated geological model may be rejected on the basis of any one of the geological tests, in which case the proposed change is discarded and the inversion commences a new iteration at Step 3a. At Steps 3c and 3d, each of the requested geophysics fields are computed in turn, and their likelihood evaluated. If the changed model is rejected on the basis of a computed likelihood, the proposed change is discarded and the inversion commences a new iteration at Step 3a. If the changed model is accepted on the basis of the computed likelihood for all requested geophysics fields components, the proposed change is retained, and this new model is stored. The inversion returns to Step 3a and continues to iterate around this loop. An ensemble of geological and associated property models that satisfy geological constraints and can satisfactorily explain the geophysical signature are (partially) explored by continuing this process for several million iterations.

During the initial part of the Iteration Phase, the data misfit for each field of the current model follows a generally decreasing trend. As the data misfit reaches values more in keeping with the specified data uncertainty values, the rate of change in misfit decreases, and we begin to store the models. By using a Metropolis acceptance test (Metropolis et al. 1953), these stored models are an exploration of the higher posterior probability regions of the set of acceptable *a priori* geological models.

3.4 Phase 4 – Analysis Phase

Using this procedure, it is possible to generate a large number of geological models that reproduce the gravity and magnetic observations to an acceptable degree. These models are analysed statistically. For example, the 'most probable lithology' for any voxel can be found by examining the ensemble and determining which lithology was assigned to this voxel more often than any other. A composite 'most probable' model can be compiled by combining the results thus obtained for each separate voxel into a single geological model. The various statistical outputs can be presented as voxel-models, cross-sections, and as horizontal slices. These displays can be combined with the reference geological model to highlight regions of concordance and discordance between the reference model and the statistical products of the inversion.

4 Application to the Bet Bet Region

A 3D geological model of the Bet Bet region, Victoria, Australia, was built from a combination of field mapping data, drillholes and interpretive inferences (Fig. 1a). The 33 km E/W by 57 km N/S by 7.7 km vertical extent project volume was discretised into 33 x 57 x 77 voxels having dimensions of 1000 m E/W, 1000 m N/S, and 100 m vertically (Fig. 1b).

Eleven geological units were used in the model. For several of these, only limited physical property data were available and we were forced to determine suitable property values by alternate means. The method used to select these values was underpinned by the premise that the geometry for these units in the reference model was approximately correct. This is not unreasonable given limited knowledge for the geological units in question. First, we isolating the region occupied by each geological unit in the reference model. Unit-property response grids were then calculated for each of the geophysical fields and each of the geological units. A bounded least-squares optimisation of a set of formation weights was performed to minimise the misfit between the summed weighted formation signatures to the observed data. The bounds were set using background knowledge of likely property values for the lithological mixtures present in each geological unit. The resultant weights were interpreted as estimates of the optimal physical property values for each geology formation assuming that the reference geometry was correct. These 'optimised' values were used in the inversion work for the formations with limited *a priori* property values.

In terms of geological constraints, the lithology was assumed to be known perfectly ('fixed') for those voxels coinciding with areas of mapped outcrop, and also for voxels pierced by drillholes. The inversion was required to maintain an approximate overlap of 80% in the distribution of each unit relative to the configuration in the reference geological model using the 'commonality' geological constraint. This implies that the reference model was considered to be 'approximately 80% correct'. The shape for each unit was also generally maintained to produce shapes that were within 5% of the shape expressed in the reference mode using the 'shape ratio' geological constraint.

We confirmed through forward modelling that the geophysical response grids associated with the reference geological model reproduced the first order features of the observed total magnetic intensity and vertical gravity gradient data. Several inversion runs were then carried out, generating many tens of millions of stored models in a procedure that took several days to run on a single desktop PC. Through statistical analysis, elements of the geometry that were common to many of these models were identified (Fig. 1c). A revised 3D geological model was build by introducing secondary geological constraints at locations where these common features were significantly different to the original reference configuration (Fig. 1d). The revised 3D geological model was consistent not just with the geological observations but also with the observed gravity and magnetic data. Although not shown, a 3D probability map was produced for each of the geological units to communicate the degree of *a posteriori* uncertainty in the mapping.

5 Recent and planned developments

The stochastic litho-inversion method described in this paper has been implemented in the GeoModeller software package (GeoModeller, 2007). The release of version 1.2 of the software in July 2007 marked the culmination of a 30 month development and commercialisation phase by Intrepid Geophysics and BRGM, supported by a number of government agencies and companies.

The functionality of the software is the subject of ongoing research and development, and the advances in the last year have included development of a process model with a formal schema for forward and inverse geophysical computation, the addition of support for full tensor gradiometry, and improvements to the voxel-based statistical reporting/presentation of inversion results.

There is always an aspiration to represent more complex and realistic geology problems and this requires finer discretisation of the geological model, and hence an increase in the total number of voxels, possibly by an order of magnitude or more. This in turn means that it will take a larger number of iterations to obtain a reasonable sample of the higher posterior probability geological models. A cluster-computer approach that will allow a user to generate a solution for such tasks within an acceptable timeframe is planned.

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