EXTENDED ABSTRACT SUBMISSION

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AN EXTENSION OF THE CLOSED-FORM SOLUTION FOR THE GRAVITY CURVATURE (BULLARD B) CORRECTION IN THE MARINE AND AIRBORNE CASES

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ABSTRACT

Geoscience Australia recently revised the corrections applied to the Australian National Gravity Database (ANGD) and switched from applying the simple Bouguer correction to the observed gravity values in its database to applying the more accurate gravity curvature (Bullard B) correction. This change is a straightforward procedure in the case of land-based gravity surveys. However, due to the inherent non-linearity of the Bullard B correction, the original formula for the gravity curvature correction is not applicable to observed data from gravity surveys, which involve layers of different materials, as is the case in marine or airborne gravity surveys. Here we present an extension of the closed-form solution for the Bullard B correction, which allows its proper application in any gravity survey setting. In particular, we present formulae to correctly apply the Bullard B correction to observed gravity data from airborne and marine gravity surveys.

INTRODUCTION

After completion of a gravity survey numerous corrections need to be applied to the raw data. In geophysics the objective of these corrections is to remove a standard earth model and to leave behind the anomalies that are due to density variations in the Earth's interior (Li & Goetze, 2001; Chapin, 1996). Usually, each correction is subtracted from the measured value, e.g. the theoretical gravity of the ellipsoid, the free air correction, and others (Blakely, 1996, p. 128ff.). One critical correction in this process is the simple Bouguer correction, which takes the gravity effect of the material between geoid (or ellipsoid) and observer into account. It is calculated assuming an infinite slab with constant density and thickness equal to the elevation of the gravity station.

The simple Bouguer correction is linear and surprisingly does not depend on the height of the gravity station above the infinite slab (c.f. Telford et al., 1990, p. 37ff). Thus, the correction only depends on the material density and the thickness of the layer. For a land-based gravity station, the simple Bouguer correction Δg_{sb} to a gravity measurement is simply given by

(1) $$\Delta g_{sb} = 2\pi\gamma\rho_{cr} h,$$

where γ is the gravitational constant, ρ_{cr} the density and h the thickness of the infinite Bouguer slab, i.e. the height of the gravity station above the geoid (or ellipsoid). In the case of a land-based survey, the density of the infinite Bouguer slab is usually chosen to be the average value of crustal material, namely $\rho_{cr} = 2.67$ t/m^3.

Due to the properties of the simple Bouguer correction, it is easy to derive the correction for very different survey settings, i.e. land-based, airborne, marine surveys, on ice sheets, etc. For instance, the simple Bouguer correction term for an airborne survey is given by

(2) $\Delta g_{sb} = 2\,\pi\gamma\rho_{cr}\,(h\text{-}d)$,

where h is the height of the aircraft above the geoid (or ellipsoid) and d it's clearance, i.e. the actual flying height above the surface. Since the gravity induced by an infinite slab does not change with height above the slab, the actual flying height of the aircraft is irrelevant and the difference (h-d) is the thickness of the infinite Bouguer slab whose gravity has to be subtracted from the gravity measurement.

It is important to remember a frequently encountered inconsistency in the correction of gravity measurements. Some corrections applied to the raw gravity measurements, such as the theoretical gravity, use the ellipsoid as reference surface whereas the geoid is often used as reference surface for e.g., the simple Bouguer correction and others. Mostly, this is done to simplify the reduction of the gravity data. For regional gravity surveys the error introduced by the use of two different reference surfaces is approximately constant over the survey area and simply results in a DC shift of the data. However, this is not the case for large-scale surveys (c.f. Blakely, 1996) and the difference between the geoid and the ellipsoid at each gravity station has to be taken into account.

To remove this potential source of confusion, Geoscience Australia recently revised the corrections applied to the Australian National Gravity Database (ANGD) and adopted the ellipsoid as reference for all corrections that are applied to the observed gravity values in the database (Tracey et al., 2008). Thus, all elevations have to be converted to heights above ellipsoid: Given the elevation H of a gravity station with respect to the geoid, the height above the ellipsoid is simply h=H+N, where N is the geoid-ellipsoid separation (positive if the geoid lies above the ellipsoid, c.f. Li & Goetze, 2001).

The revised ANGD corrections also exchanged the simple Bouguer correction for the Bullard B correction, which takes the curvature of the earth into account. The Bullard B correction replaces the infinite slab of the simple Bouguer correction by a spherical cap of equal thickness and a radius of 166.735 km (Bullard, 1936). However, the spherical cap approximation is highly non-linear and, contrary to the simple Bouguer correction, does depend on the observation height above the spherical cap.

AN EXTENSION OF THE CLOSED-FORM SOLUTION OF THE BULLARD B CORRECTION

A closed form of the Bullard B correction was published by LaFehr (1991) for the case of land-based observations. Here, we present an extension of this closed-from solution, which is not only valid for land-based observations but which can be applied to any gravity survey setting, such as marine or airborne. The extension is based on calculating the z-component of gravity, $g_z(S_0,t)$, for a point P at height t above the spherical cap of radius S_0 closing a cone with angle 2α subtended at the earth's center, using Newton's law of gravity (see Fig. 1). The actual calculation of $g_z(S_0,t)$ at P is lengthy but straightforward and will not be shown here. However, the full result is given in the appendix, together with a reproduction of LaFehr's original formula for comparison purposes. Note, that $\alpha = R_b/R_0$, where R_b=166.735 km is the Bullard B surface radius (Bullard, 1936; LaFehr, 1991) and R_0=6371.0087714 km is the mean radius of the earth, based on the GRS 1980 value from Moritz, 1980.

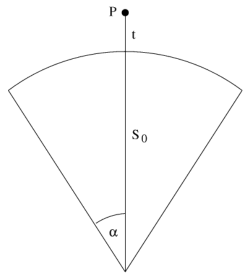

Figure 1: P is the location of the gravity station above the cone, t is the distance of P from the spherical cap closing the cone, S_0 is the radius of the spherical cap, and 2α is the opening angle of the cone subtended at earths center (see text).

Fig. 1 shows a sketch of the cone and the location of the gravity station above its spherical cap. The z-component of gravity induced by the cone at the observation point P is given by

(3) $$g_z(S_0,t) = 2\,\pi\gamma\rho_{cr}\,(\,(S_0 + t)\,[1 + \lambda'(S_0,t) - \kappa] - |t|\,[1 + \mu(S_0,t)]\,).$$

The dimensionless quantities $\mu(S_0,t)$, $\lambda'(S_0,t)$ and κ are defined in the appendix. They have been defined in such a way that they are as close as possible to the corresponding terms given by LaFehr (1991). Note, that Eq. 5 is also valid for $t<0$, i.e. for a subsurface gravity station.

Given Eq. 5, it is now easy to calculate the Bullard B correction Δg_{bb} in any gravity survey setting as the difference between the gravity induced by two or more cones with different radii and heights of the observer above the spherical cap. For instance, the Bullard B correction for a land-based gravity station at elevation h above the ellipsoid is given as the difference between a cone with radius $S_0=(R_0+h)$ and gravity meter elevation t=0, and a cone with radius $S_0'=R_0$ and gravity meter elevation t'=h.

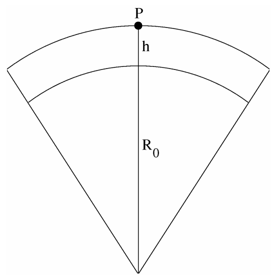

Figure 2: Sketch of the relevant spherical cones for the Bullard B correction in the land-based case. P is the location of the gravity station, R_0 is the mean radius of the earth and h is the elevation of the gravity station above the ellipsoid. The segment below P is the spherical cap for which we wish to know the gravity on its surface at its center.

Following this scheme, the Bullard B correction can easily be calculated for any other gravity survey setting. In particular, we have the following cases:

- Land-based (c.f. Fig. 2)

  (4) $$\Delta g_{bb} = g_z(R_0+h,0) - g_z(R_0,h)$$

  which reduces to the formula presented by LaFehr (1991).

- Airborne (c.f. Fig. 3)

  (5) $$\Delta g_{bb} = g_z(R_0+h-d,d) - g_z(R_0,h),$$

where h is the height of the aircraft above the ellipsoid and d its clearance, i.e. its actual flying height above the surface.

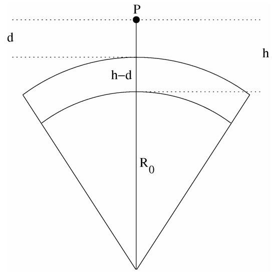

Figure 3: Same as Fig. 2 but for the airborne case. h is the height of the aircraft above the ellipsoid and d is the clearance, i.e. the actual flying height above the surface.

- Marine (c.f. Fig. 4)

(6) $\Delta g_{bb} = g_z^{(\rho cr-\rho sw)}(R_0+N,0) - g_z^{(\rho cr-\rho sw)}(R_0+N-D,D) \pm [g_z^{\rho cr}(R_0,N) - g_z^{\rho cr}(R_0+N,0)]$

where D>0 is the ocean depth measured from the ocean surface, N the ellipsoid-geoid separation and $g_z^{(\rho cr-\rho sw)}(.,.)$, $g_z^{\rho cr}(.,.)$ denote the formula given in Eq. 3 but using the difference between crustal and sea-water (ρ_{cr}-ρ_{sw}) and crustal ρ_{cr} densities, respectively. The plus sign has to be used for the last term if N is positive (geoid above ellipsoid) and the minus sign if N is negative (geoid below ellipsoid). Fig. 4 illustrates the case N>0, for N<0 the position of the ellipsoid and geoid have to be switched.

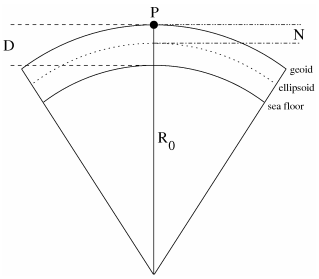

Figure 4: Same as Fig. 2 but for the marine case. Indicated are the position of the sea floor, the ellipsoid and the geoid. D is the depth and N the geoid-ellipsoid separation. Note that the relative position of the geoid and ellipsoid in this figure are switched if N<0, which results in a sign reversal of the correction term (see text).

DISCUSSION

For airborne surveys, the extended spherical cap correction takes the attenuation of the gravity field with height above the spherical cap into account. This is similar to the traditional free air correction, which compensates for the attenuation of gravity above the ellipsoid. Here, however, the inhomogeneous gravity field of the additional mass above the ellipsoid is accounted for. This is of no concern when using the simple Bouguer correction, since the gravity field above an infinite plane is constant, i.e. does not depend on the distance of the observer from the plane. This additional free-air correction is of the order <1-1.5 μm/s^2 for a flying height of 100 m above ground. The attenuation of the gravity field will obviously increase for larger flying heights above the ground.

In the case of a marine survey setting, the last term of Eq. 6 describes the gravity field of the section between the geoid and ellipsoid, which is devoid of or filled by sea-water. Geometrically, the observer is located on top of the spherical section if N>0, whereas she is underneath the spherical section if N<0. This will result in a slightly different Bullard B correction in these two cases, whereas the simple Bouguer correction will have the same value for both N>0 or N<0. Again, the inhomogeneous gravity field of the spherical cap is responsible for this distinct behavior of the Bullard B correction.

In all the gravity survey settings considered here, the difference between the Bullard B and the simple Bouguer correction is generally of the order of 10-20 $\mu m/s^2$, which is by no means negligible for high-precision gravity surveys.

CONCLUSIONS

The simple Bouguer correction is a linear function of the density and the thickness of a sheet of matter. Thus, it is possible to simply add the gravity effects from sheets of matter with different densities. This is done, for example, when calculating the simple Bouguer correction for a marine survey. Contrary to the infinite slab used in the simple Bouguer correction the gravity field induced by the spherical cap of the Bullard B correction is not homogeneous. This results in highly non-linear properties of the Bullard B correction and a simple addition of the gravity of two sheets of matter is no longer possible. The presented closed-form solution of the Bullard B correction takes these non-linear effects into account and should be used for high-precision gravity surveys.

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APPENDIX – THE EXTENDED CLOSED-FORM SOLUTION OF THE GRAVITY CURVATURE (BULLARD B) CORRECTION

The Bullard B correction for a point at the center of a spherical cap with thickness t and radius S_0 above a cone with angle 2 α subtended at the earth's center was given by LaFehr (1991) as

$$\Delta g_{bb} = 2\pi\gamma\rho \; ((1+\mu) \; t - \lambda(S_0+t)),$$

where γ is the gravitational constant and ρ the average crustal density. Furthermore, the following definitions are used:

$\mu = 1/3 \; \eta^2 - \eta,$
$\lambda = 1/3 \; [\; (d+f\delta+\delta^2) \; ((f-\delta)^2+k)^{1/2} + p + m \ln(\; n \, / \, ((f-\delta) + ((f-\delta)^2+k)^{1/2}) \;) \;],$
$\eta = t/(S_0+t),$
$\delta = S_0/(S_0+t),$

where $d=3 \cos^2\alpha-2$, $f=\cos\alpha$, $k=\sin^2\alpha$, $p=-6 \cos^2 \alpha \; \sin(\alpha/2)+4 \sin^3(\alpha/2)$, $m=-3 \sin^2\alpha \cos \alpha$, $n = 2[\sin(\alpha/2) - \sin^2(\alpha/2)]$, and $\alpha=R_b/R_0$ (see text).

The extended closed form solution of the Bullard B correction uses the gravity $g_z(S_0,t)$ at a point P at height t above or below a spherical cap of radius S_0 closing a cone with angle 2α subtended at the earth's center (see Fig. 1). It is given by

$$g_z(S_0,t) = 2 \; \pi\gamma\rho \; (\; (S_0+t)[1+\lambda'(S_0,t) - \kappa] - |t| \; [1 + \mu(S_0,t)] \;).$$

The equation uses most of the definitions above, together with

$\lambda'(S_0,t) = 1/3 \; [\; (d+f\delta+\delta^2) \; ((f-\delta)^2+k)^{1/2} - m \ln(\; (f-\delta) + ((f-\delta)^2+k)^{1/2} \;) \;],$
$\kappa = 1/3 \; (d + 2 - m \ln(b) \;),$ and
$b = 2 \cos^2(\alpha/2).$

Note, that $\lambda'(S0,t)$ is defined slightly different to the λ defined by LaFehr (1991) and that the dimensionless quantities $\mu(S0,t)$ as well as $\lambda'(S0,t)$ are not constants but depend on S0 and t.