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FURTHER DEVELOPMENTS WITH FULL TENSOR GRADIOMETRY DATASETS

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ABSTRACT

With the advent of new potential field full tensor gradient instrumentation, new methods have been developed to denoise and process these curvature gradients. Traditional Fourier Domain and Minimum Squares least squares residual of the linear differential relationships have been adapted. This leads to levelling, gridding and grid filtering innovations. The result is a Full Tensor Grid representation of the curvature gradients that is coherent and compliant with the physics at all points in the grid. All of the observed data is thus honoured in the Tensor grid. Superior anomaly interpretation and inferences can then be made. A case study showing the improvement that can be obtained is presented. Special attention is warranted for the Full Tensor Magnetic gradient signal. Multiple surveys of this quantity have been made recently in South Africa.

Key Words: Potential Fields – Processing Methods, Tensors.

INTRODUCTION

The realisation that newer generation potential field survey measurements cannot be treated as scalars has taken time to be accepted. FitzGerald (2006) develops the case for using an 'observed' data object that is a container of an arbitrarily complex observation. This covers the full scale from simple magnitude through to Full Tensor Gradiometry (FTG). Vassiliou (1985) discusses appropriate Fourier methods for grid based transforms, including integration. Pedersen & Rasmussen (1990) go through simple transforms and properties of the tensor to bring out interpretation possibilities. Among the many studies that model the possibilities for measuring an airborne FTG signal to use as an exploration tool, Dransfield (1994) is one of the first. The aim here is to concentrate on FTG and to record many methods and developments that have been shown to work in practice on large scale survey datasets, in a manner suitable for everyday use.

There are several advantages both computationally and from a signal processing / interpretation point of view to looking past a differential calculus treatment of full tensor gradients to an Amplitude / Phase Treatment. The basic idea follows from a realization that the potential fields under consideration are stationary and instantaneous at each point in 3D space. At each observation point, there are three well known tensor invariants which are constant under rotation of the co-ordinate reference frame of the observer. They can be obtained by a simple principal-component analysis of the symmetric tensors encountered in geophysical applications.

We decompose each FTG reading into the invariant eigenvalue amplitudes along with the rotation matrix local to the survey reference frame. The rotation matrix represents the 'phase' of the signal. Of course, it is always possible to state the tensor in either traditional or the amplitude / phase notation at any time. We term our process a ‘separation of concerns’. It opens up many opportunities for development in methods and understanding.

The issues discussed cover many aspects of potential field practice. The work has continued for many years. Not all aspects can be proven in an exact scientific sense, but against this a considerable body of field observations has been used to test the ideas and methods. A subset of these test datasets together with batch procedures that capture and demonstrate reproducibility of each process is freely available from the authors.

THE AMPLITUDE / PHASE TREAMENT OF TENSORS

Tensors encountered in FTG applications take the form of rank three symmetric matrices with zero trace. Thus, they possess just five out of nine independent components, namely G_{xx}, G_{xy}, G_{xz}, G_{yy}, and G_{yz}. The theory of scalar potentials tells us that the sixth component, G_{zz}, can be found by exploiting Laplace's equation, i.e. $G_{zz} = -(G_{xx} + G_{yy})$. The remaining three components are determined from symmetry. Principal component analysis is then applied to decompose the tensor into a diagonal matrix containing the eigenvalues, which are independent of the local reference frame, and an orthogonal rotation matrix with the associated eigenvectors depending on the observer’s local reference frame.

The easiest way to handle rotations consistently is by transforming them into so-called unit quaternions. Quaternions were discovered by Hamilton (1853) and provide a very powerful method of parameterising rotations. Today quaternions are extensively used in computer graphics, particularly in 3D animation. There is a vast set of references covering this development. Importantly, a property of rotations, and thus quaternions, is that they are non-commutative under multiplication, i.e. $a \cdot b \neq b \cdot a$. This means we must take care in every mathematical manipulation of the data. This decomposition of the tensor into an invariant, structural part (eigenvalues) and rotational part (eigenvectors) paves the way to fast and robust processing of tensor data which respects the intrinsic physical properties of tensors.

GRIDDING METHODS

The estimation of gradient tensors at regular grid intervals away from observed profiles is an essential requirement for FTG signal processing, and as yet, a little studied subject. One technique that can be used to create grids of tensor components that look ‘smooth and coherent’ is the Minimum Curvature algorithm, developed by Ian Briggs (1974) of the Bureau of Mineral Resources (now known as Geoscience Australia). It has become a standard gridding technique, not just for potential fields.

Unfortunately, processing tensor data on a component-by-component basis bears the danger of corrupting the signal. Handling tensor data in this way makes no attempt to preserve the Laplace condition, i.e. the defining physical characteristic of tensors derived from a scalar potential. As will be seen below, this is especially noticeable in regions of strong curvature of the potential field, or in other words, where the geological features causing the field anomalies are located. Thus, an interpolation method for tensors is required which takes their intrinsic physical properties into account.

Following the separation of concerns into amplitude and rotations, it follows to seek a means for smoothly interpolating rotations and to combine this with amplitude estimation. The key is to realize that the universal covering group of the group of rotations in 3D space, SO(3), is given by the group of unit quaternions, S^3. In other words, unit quaternions define a hypersphere and each point on this hypersphere defines a rotation axis and a rotation angle. Note, that this is a double covering: For each quaternion q, there is a second quaternion, namely -q, which describes the same rotation. Our task is now to interpolate between two points on the hypersphere along the shortest path between them. Thus, interpolated points lie along geodesics of S^3, which are given by great circles, the analogue of line segments in the plane. The interpolation of points along great circles is achieved by so-called spherical linear interpolation, or "SLERP" (Shoemake 1995; Kuipers 1999). Since the structural part of the tensor, i.e. the matrix of eigenvalues, has been separated from the rotational part, standard minimum curvature algorithms can be applied for its interpolation. Currently, full tensor interpolation is implemented in the Intrepid software as a triangle or 3-point interpolation algorithm. There is a patent pending on this development. (Ref: Australian Patent Application No. 2006900346 in the name of Desmond FitzGerald & Associates Pty Ltd "An improved method of interpolation between a plurality of observed tensors").

Fig. 1 illustrates that significant corruption of the signal can occur when processing tensors on a component-by-component basis. The figure shows the Laplacian Gxx+Gyy+Gzz of tensor data over a geological feature, which should always evaluate to zero. This is not true for data that has been treated on a component-by-component basis (left panel). A strong correlation with geology is easily apparent. Using SLERP interpolation (right panel) only shows numerical noise, indicating that the Laplacian relationship is always honoured.

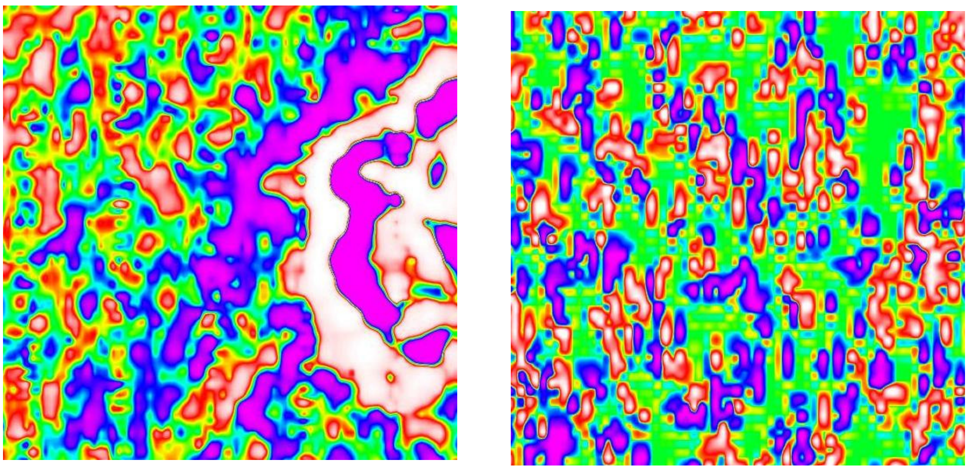

Figure 1: The trace of tensors should always be zero to honour the Laplacian relationship. Processing on a component-by-component basis (left panel) shows a clear correlation of the trace with geology. Interpolating tensors using quaternion SLERP only shows numerical noise.

LEVELLING METHODS

The most popular FTG levelling method currently deployed is a variation on a 'heading' correction. This is often used in a way that cannot be justified scientifically. It is reasonable to apply the method once, but not 40 or 50 times, as sometimes done in practice. The reason that in-line and cross-line, as well as tensor data is subjected to this excessive 'correction' is the lack of an alternative approach. Using the Amplitude / Phase treatment of tensors, it is possible to define a heading correction that works similarly to the well-known heading correction for scalar data. All that is needed is to define a tensor average that preserves the intrinsic physical properties of tensors. One way to achieve this is to calculate the arithmetic mean of the eigenvalues and to employ Fisher statistics (Fisher, 1953) to determine the average rotational part. After calculation of the average tensor along flight and tie lines, the heading correction can then be applied in the same manner as the traditional algorithm. Alternatively, Pajot et al. (2007) detail grid based methods that use linear differential relationships to level or denoise FTG data.

Furthermore, we examined perhaps the simplest of levelling procedures from surveying – the 'Loop Closure' method (Green 1983). This is a least squares minimisation technique that works very well in weak gradient situations. The requirement of a least squares solution is that the sum of the observed differences around every intersection be equal to the sum obtained from the estimated values. In implementing this method for FTG, the Frobenius Norm of the delta misclosure tensor at each crossover point was chosen as the function to minimise. This method works quickly and efficiently and follows the original philosophy exactly. It should be applied after all obvious "corrugations" have been previously levelled, e.g. with methods similar to the heading correction discussed above. Fig. 2 demonstrates the improvements that can be achieved when adding both heading and loop corrections to actual magnetic FTG data.

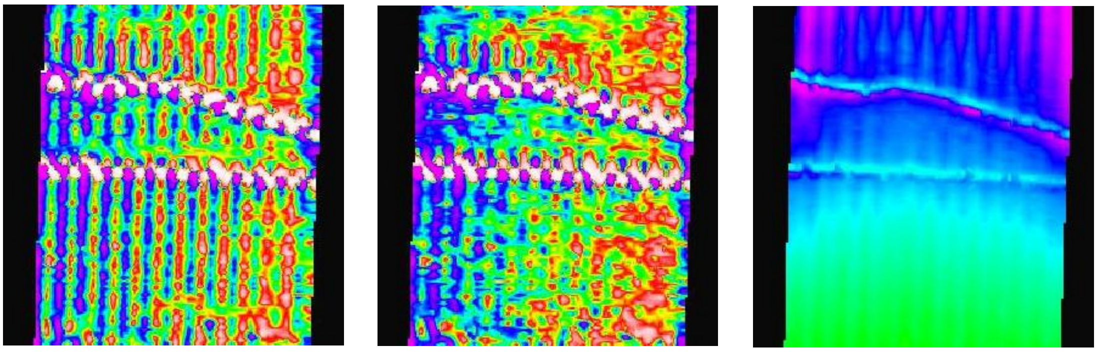

Fig. 2: Raw magnetic FTG data (left panel), data after heading correction (middle panel), and after loop correction using 10 iterations. The obvious structures bisecting the figures are two dikes. Note, that there is no tie line at the top of this survey.

Although levelling corrections have the potential to considerably improve FTG data, there is a possible source of levelling errors when processing magnetic data obtained with SQUID devices. Both during a flight and between flights, "flux jumps" in the SQUID device can occur. These manifest as abrupt simultaneous jumps in the measured raw gradients in usually two or more of the measuring channels. The explanation is offered that the measurement electronics is ideally working in a linear stable portion of the cyclical flux, but occasionally jumps a cycle to a new stable position.

The method of detection and removal follow similar lines to 4^{th} difference spike detection and removal. Tares in the meter reading between flights are also very common. No good explanation is offered other than the above "flux jump" settling to a new equilibrium position. This requires a flight-based adjustment of the average tensor for each flight. It raises the issues of which average flight tensor is the one to choose to adjust the others. Intuitively, for the magnetic tensor, the one closest to zero gradient, or quietest magnetically, should be chosen.

FOURIER METHODS

Fourier methods play an important role in signal processing and the Amplitude / Phase treatment of tensors immediately lends itself for efficient Fourier methods. In particular, there is the recent discovery of a Fast Fourier Transform (FFT) of quaternions (Moxey et al., 2002) that uses two complex numbers to represent quaternions. Together with two FFTs for two of the three eigenvalues, the Fourier transform of a tensor can be contained by four complex Fourier transformations, instead of five when processing tensors on a component-by-component basis. Standard filtering operations that respect the inherent physical properties of the data have been implemented for line or grid-based FTG data, including low-pass, high-pass, band-pass filters and others. Fig. 3 shows a low-pass filter applied to line-based data from a magnetic FTG survey.

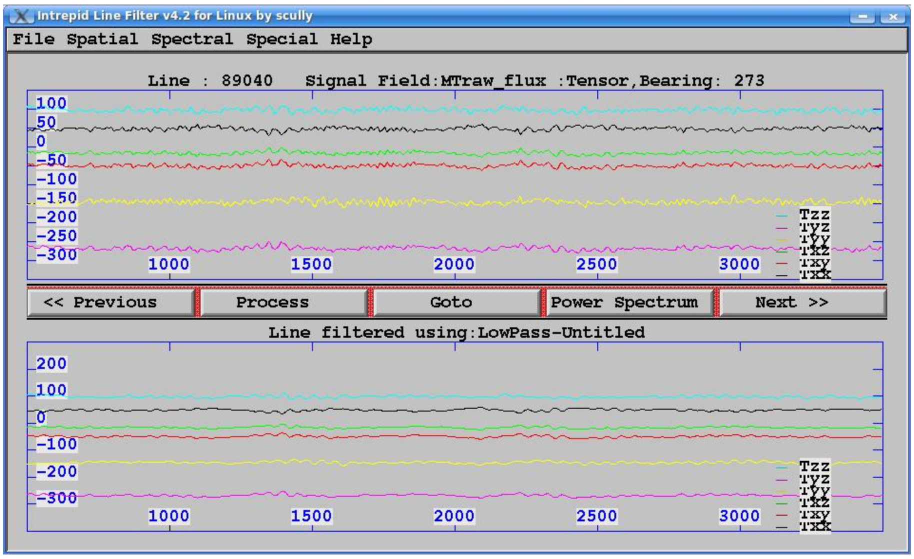

Fig. 3: Low-pass filter applied to magnetic FTG data. The upper half shows the original noisy data, the lower half shows the low-passed signal.

CONCLUSIONS

The methods outlined in this paper have proven to be very useful both for signal processing, gridding and interpretation of FTG data. They require the correct treatment of (non-commutating) tensors as compound objects without resorting to process the components separately. This is a direct result of recognizing the rotational components within the signal. Both magnetic and gravity full tensor gradiometry surveys have been processed using this fast and robust approach. A standard laptop computer can routinely be used in the field to process several flights worth of data from a day’s flying in less than one hour. The most intensive part of the work is the use of FFT methods to decimate and filter the signal to create the located geophysical database. A set of standardised, repeatable tests of all the methods is captured in both model and observed FTG datasets. This is available to those interested in investigating and becoming better acquainted with the methods (www.intrepid-geophysics.com).

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